SECTION A – Linear Dynamics

1976B1
a.

b. \( \Sigma F = ma; \ T - W - 2F_f = 800 \text{ N}; \ T = 5000 \text{ N} \)
c. Looking at the FBD for the counterweight we have \( \Sigma F = ma; \ Mg - T = Ma \)
   \[ M = \frac{T}{(g - a)} \] where \( T = 5000 \text{ N} \) gives \( M = 625 \text{ kg} \)

1979B2
a. \( \Sigma F = ma; \ 50 \text{ N} - f = ma \) where \( f = \mu N \) and \( N = mg \) gives \( 50 \text{ N} - \mu mg = ma; \ a = 3 \text{ m/s}^2 \)
b.

c. \( \Sigma F = ma \) for each block gives \( W_5 - T = m_5a \) and \( T - f = m_{10}a \). Adding the two equations gives \( W_5 - f = (m_5 + m_{10})a \), or \( a = 2 \text{ m/s}^2 \)

1982B2
a.

b. \( T_1 \) is in internal system force and will cancel in combined equations. Using \( \Sigma F_{\text{external}} = m_{\text{total}}a \) gives \( T_2 - m_1g - m_2g = (m_1 + m_2)a \), solving yields \( T_2 = 6600 \text{ N} \). Now using \( \Sigma F = ma \) for the load gives \( T_1 - m_1g = m_1a \) and \( T_1 = 6000 \text{ N} \)
1985B2
a. Note that the system is at rest. The only forces on the hanging block are gravity and the tension in the rope, which means the tension must equal the weight of the hanging block, or 100 N. You cannot use the block on the incline because friction is acting on that block and the amount of friction is unknown.
b. 
\[ \Sigma F = 0; f_s + mg \sin \theta - T = 0 \] gives \( f_s = 13 \) N
c.

1986B1
a. \( \Sigma F_{external} = m_{total}a; m_4g - m_1g - m_2g = (m_4 + m_2 + m_1)a \) gives \( a = 1.4 \) m/s\(^2\)
b. For the 4 kg block:
\[ \Sigma F = ma \]
\[ mg - T_4 = ma \]
gives \( T_4 = 33.6 \) N
c. Similarly for the 1 kg block: \( T_1 - mg = ma \) gives \( T_1 = 11.2 \) N

1987B1
a.
b. \( \Sigma F_{external} = m_{total}a; \) Where the maximum force of static friction on mass \( M_1 \) is \( \mu_s N \) and \( N = M_1g; M_2g - \mu_s M_1g = 0 \) gives \( \mu_s = M_2/M_1 \)
c/d. \( \Sigma F_{external} = m_{total}a \) where we now have kinetic friction acting gives \( M_2g - \mu_k M_1g = (M_1 + M_2)a \)
so \( a = (M_2g - \mu_k M_1g)/(M_1 + M_2) \)
\[ \Sigma F = ma \] for the hanging block gives \( M_2g - T = M_2a \) and substituting \( a \) from above gives \( T = \frac{M_1 M_2 g}{M_1 + M_2} (1 + \mu_k) \)

1988B1
a. 
b. \( \Sigma F = ma \) gives \( T - mg = ma \) and \( T = 1050 \) N
c. The helicopter and the package have the same initial velocity, 30 m/s upward. Use \( d = v_i t + \frac{1}{2} at^2 \)
\[ d_h = (+30 \text{ m/s})t + \frac{1}{2} (+5.2 \text{ m/s}^2)t^2 \] and \( d_p = (+30 \text{ m/s})t + \frac{1}{2} (-9.8 \text{ m/s}^2)t^2 \).
The difference between \( d_h \) and \( d_p \) is 30 m, but they began 5 m apart so the total distance is 35 m.
1998B1

a. \( \Sigma F_{\text{ext}} = m_{\text{tot}}a \) gives \( mg = 2ma \), or \( a = g/2 \)

b. \( d = v_0t + \frac{1}{2}at^2; \ h = 0 + \frac{1}{2} \left( g/2 \right)t^2 \) gives \( t = 2 \sqrt{\frac{h}{g}} \)

c. Block A accelerates across the table with an acceleration equal to block B \( (g/2) \).

d. Block A is still in motion, but with no more applied force, Block A will move at constant speed across the table.

e. Since block B falls straight to the floor and stops, the distance between the landing points is equal to the
horizontal distance block A lands from the edge of the table. The speed with which block A leaves the tabletop
is the speed with which block B landed, which is found from \( v = v_0 + at = \frac{g}{2} \left( 2 \sqrt{h/g} \right) = \sqrt{hg} \) and the time for
block A to reach the floor is found from \( 2h = \frac{1}{2}gt^2 \), which gives \( t = 2 \sqrt{\frac{h}{g}} \).

The distance is now \( d = vt = \sqrt{hg} \times 2 \sqrt{\frac{h}{g}} = 2h \)

2000B2

a. 

b. \( f = \mu N \) where \( N = m_1g \cos \theta \) gives \( \mu = \frac{f}{m_1g \cos \theta} \)

c. constant velocity means \( \Sigma F = 0 \) where \( \Sigma F_{\text{external}} = m_1g \sin \theta + m_2g \sin \theta - f - Mg = 0 \)
solving for \( M \) gives \( M = (m_1 + m_2) \sin \theta - (\frac{f}{g}) \)

d. Applying Newton’s second law to block 1 gives \( \Sigma F = m_1g \sin \theta - f = m_1a \) which gives \( a = g \sin \theta - f/m_1 \)

2003B1

a. 

b. The tension in the rope is equal to the weight of student B: \( T = m_Bg = 600 \) N
\( \Sigma F_A = T + N - m_Ag = 0 \) gives \( N = 100 \) N

c. For the climbing student \( \Sigma F = ma; \ T - m_Bg = m_Ba \) gives \( T = 615 \) N

d. For student A to be pulled off the floor, the tension must exceed the weight of the student, 700 N. No, the
student is not pulled off the floor.

e. Applying Newton’s second law to student B with a tension of 700 N gives \( \Sigma F = T - m_Bg = m_Ba \) and solving
gives \( a = 1.67 \text{ m/s}^2 \)

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2003Bb1

a. 

b. We can find the acceleration from \( a = \Delta v/t = 2.17 \text{ m/s}^2 \) and use \( d = \frac{1}{2} at^2 \) to find \( d = 975 \text{ m} \)

c. The x and y components of the tension are \( T_x = T \sin \theta \) and \( T_y = T \cos \theta \) (this is using the angle to the vertical). Relating these to the other variables gives \( T \sin \theta = ma \) and \( T \cos \theta = mg \). Dividing the two equations gives \( \tan \theta = \frac{a}{g} = \frac{(2.17 \text{ m/s}^2)}{(9.8 \text{ m/s}^2)} \) and \( \theta = 12.5^\circ \)

1996B2

a. There are other methods, but answers are restricted to those taught to this point in the year.

i. A device to measure distance and a calibrated mass or force scale or sensor

ii. Hang the mass from the bottom of the spring and measure the spring extension (\( \Delta x \)) or pull on the spring with a known force and measure the resulting extension.

iii. Use Hooke’s law with the known force or weight of the known mass \( F = k \Delta x \) or \( mg = k \Delta x \) and solve for \( k \)

b. Many methods are correct, for example, place the object held by the scale on an inclined plane and find the weight using \( W \sin \theta = k \Delta x \). One could similarly use a pulley system to reduce the effort applied by the spring scale.

2007B1

a. \( x = vt \) gives \( t = \frac{(21 \text{ m})}{(2.4 \text{ m/s})} = 8.75 \text{ s} \)

b. 

c. \( \Sigma F = 0 \) if the sled moves at constant speed. This gives \( mg \sin \theta - f = 0 \), or \( f = mg \sin \theta = 63.4 \text{ N} \)

d. \( f = \mu N \) where \( N = mg \cos \theta \) so \( \mu = f/N = \frac{(mg \sin \theta)}{(mg \cos \theta)} = \tan \theta = 0.27 \)

e. i. The velocity of the sled decreases while its acceleration remains constant

ii. 

2007B1B

a. 

b. \( \Sigma F_y = 0; N + T \sin \theta - mg = 0 \) gives \( N = mg - T \sin \theta = 177 \text{ N} \)

c. \( f = \mu N = 38.9 \text{ N} \) and \( \Sigma F_x = ma; T \cos \theta - f = ma \) yields \( a = 0.64 \text{ m/s}^2 \)

d. 

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1981M1

a. 

b. F can be resolved into two components: F sin θ acting into the incline and F cos θ acting up the incline. The normal force is then calculated with \( \Sigma F = 0 \); N – F sin θ – mg cos θ = 0 and f = μN. Putting this together gives \( \Sigma F = ma \); F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = ma, solve for a.

c. For constant velocity, a = 0 in the above equation becomes F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = 0. Solving for F gives \( F = \frac{mg (\frac{\mu \cos \theta \sin \theta}{\cos \theta - \mu \sin \theta})}{\} \) In order that F remain positive (acting to the right), the denominator must remain positive. That is cos θ > μ sin θ, or tan θ < 1/μ.

1986M1

a. Combining the person and the platform into one object, held up by two sides of the rope we have \( \Sigma F = ma \); 2T = (80 kg + 20 kg)g giving T = 500 N.

b. Similarly, \( \Sigma F = ma \); 2T = 1000 N = (100 kg)(2 m/s²) giving T = 600 N.

c. For the person only: \( \Sigma F = ma \); N + 600 N – mg = ma gives N = 360 N.

2007M1

a. 

b. \( \Sigma F_y = 0 \); N + F sin θ – mg = 0 gives N = mg – F sin θ.

c. \( \Sigma F_x = ma \); F cos θ – μN = ma. Substituting N from above gives \( \mu = (F \cos \theta - ma)/(mg - F \sin \theta) \)

d. 

e. The condition for the block losing contact is when the normal force goes to zero, which means friction is zero as well. \( \Sigma F_x = F_{\text{max}} \cos \theta = ma_{\text{max}} \) and \( \Sigma F_y = F_{\text{max}} \sin \theta - mg = 0 \) giving \( F_{\text{max}} = mg/(\sin \theta) \) and \( a_{\text{max}} = (F_{\text{max}} \cos \theta)/m \) which results in \( a_{\text{max}} = g \cot \theta \)
1996M2

a. \[ \Sigma F = ma; \text{ using downward as the positive direction, } mg - N = ma_y \text{ gives } N = m(g - a_y) = 2490 \text{ N} \]
b. Friction is the only horizontal force exerted; \[ \Sigma F = f = ma_x = 600 \text{ N} \]
c. At the minimum coefficient of friction, static friction will be at its maximum value \[ f = \mu N, \text{ giving } \mu = f/N = (600 \text{ N})/(2490 \text{ N}) = 0.24 \]
d. \[ y = y_0 + v_{0y}t + \frac{1}{2} a_y t^2 = 2 \text{ m} + \frac{1}{2} (-1.5 \text{ m/s}^2) t^2 \text{ and } x = x_0 + v_{0x}t + \frac{1}{2} a_xt^2 = \frac{1}{2} (2 \text{ m/s}^2) t^2, \text{ solving for } t^2 \text{ in the x equation gives } t^2 = x. \text{ Substituting into the y equation gives } y \text{ as a function of } x: y = 2 - 0.75x \]
e. 

![Graph of y vs. x](image)

1998M3

a. i. 

\[ N_1 = m_1 g \]

ii. 

\[ f_1 = 0 \]

iii. 

\[ T = M g \]

iv. 

\[ N_2 = (m_1 + m_2) g \]

v. 

\[ f_2 = M g \]

b. The maximum friction force on the blocks on the table is \[ f_{2\text{max}} = \mu_{22}N_2 = \mu_{22}(m_1 + m_2)g \] which is balanced by the weight of the hanging mass: \[ Mg = \mu_{22}(m_1 + m_2)g \] giving \[ M = \mu_{22}(m_1 + m_2) \]

c. 

For the hanging block: \[ Mg - T = Ma; \] For the two blocks on the plane: \[ T - f_3 = (m_1 + m_2)a \] Combining these equations (by adding them to eliminate \( T \)) and solving for \( a \) gives \[ a = \left[ \frac{M - \mu_{22}(m_1 + m_2)}{M + m_1 + m_2} \right] g \]
d.  
   i.  \( f_1 = \mu_{k1}m_1g = m_1a_1 \) giving \( a_1 = \frac{\mu_{k1}g}{m_1} \)
   
   ii. 

   For the two blocks: 
   
   \[
   Mg - T = M \ddot{a}_2 \quad \text{and} \quad T - f_1 - f_2 = m_2a_2
   \]
   
   Eliminating \( T \) and substituting values for friction gives 
   \[
   \ddot{a}_2 = \frac{M - \mu_{k1}m_1 - \mu_{k2}(m_1 + m_2)}{M + m_2} g
   \]

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2005M1

a. The magnitude of the acceleration decreases as the ball moves upward. Since the velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.

b. At terminal speed \( \Sigma F = 0 \). \( \Sigma F = -Mg + kvT \) giving \( v_T = \frac{Mg}{k} \)

c. It takes longer for the ball to fall. Friction is acting on the ball on the way up and on the way down, where it begins from rest. This means the average speed is greater on the way up than on the way down. Since the distance traveled is the same, the time must be longer on the way down.

d. 

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(a) [Diagram]

(b) Apply 

\[
\begin{align*}
F_{\text{net}(X)} &= 0 \\
T_P \cos 30 &= mg \\
T_P &= 20.37 \text{ N} \\
T_H &= 10.18 \text{ N}
\end{align*}
\]

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(a) [Diagram]

(b) SIMULTANEOUS EQUATIONS

\[
\begin{align*}
F_{\text{net}(X)} &= 0 \\
T_a \cos 30 &= T_b \cos 60 \\
T_a \sin 30 + T_b \sin 60 - mg &= 0
\end{align*}
\]

.... Solve above for \( T_b \) and plug into \( F_{\text{net}(y)} \) eqn and solve

\[
T_a = 24 \text{ N} \\
T_b = 42 \text{ N}
\]
a) i) \[ T = mg = 1 \text{ N} \]

ii) The horizontal component of the tension supplies the horizontal acceleration.
\[ T_h = ma = 0.5 \text{ N} \]
The vertical component of the tension is equal to the weight of the ball, as in (a) ii. \( T_v = 1 \text{ N} \)

c) Since there is no acceleration, the sum of the forces must be zero, so the tension is equal and opposite to the weight of the ball. \( T_h = \text{zero}, T_v = 1 \text{ N} \)

d) The horizontal component of the tension is responsible for the horizontal component of the acceleration. Applying Newton's second law:
\[ T_h = ma \cos \theta \]
where \( \theta \) is the angle between the acceleration and horizontal
\[ T_h = (0.10 \text{ kg})(5.0 \text{ m/s}^2) \cos 30^\circ, T_h = 0.43 \text{ N} \]
The vertical component of the tension counteracts only part of the gravitational force, resulting in a vertical component of the acceleration.
Applying Newton's second law.
\[ T_v = mg - ma \sin \theta \]
\[ T_v = (0.10 \text{ kg})(10 \text{ m/s}^2) - (0.10 \text{ kg})(5.0 \text{ m/s}^2) \sin 30^\circ, T_v = 0.75 \text{ N} \]

e) Since there is no horizontal acceleration, there is no horizontal component of the tension. \( T_h = \text{zero} \)
Assuming for the moment that the string is hanging downward, the centripetal is the difference between the gravitational force and the tension. Applying Newton's second law.
\[ mv^2/r = mg - T_v \]
Solving for the vertical component of tension:
\[ T_v = -1.5 \text{ N i.e. the string is actually pulling down on the ball.} \]